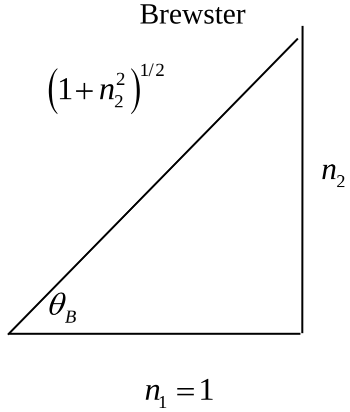
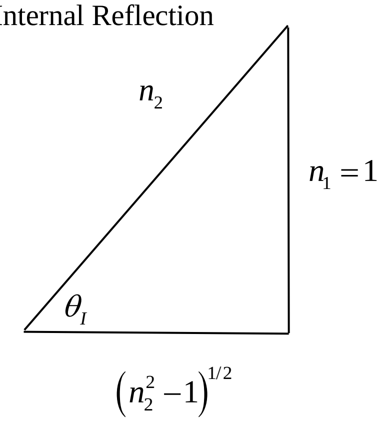
RATIONALIZATION OF THE POLARIZATION OF A RAINBOW

Reflections within the raindrops are assumed to occur at the critical angle θ_I for internal reflection, and the refractive index needed for this angle to equal the Brewster angle θ_B is then computed. This refractive index is found to be about 1.27, compared with 1.33 for water, so that partial polarization is predicted.

Brewster



Internal Reflection



The Brewster angle θ_B is given by

$$\begin{aligned}\theta_B &= \arctan\left(n_2/n_1\right) \\ &= \arctan\left(n_2\right) && \text{(for air)} \\ &= \arcsin\left[\frac{n_2}{\left(1+n_2^2\right)^{1/2}}\right] && \text{(for air)}\end{aligned}$$

and θ_I is given by

$$\begin{aligned}\theta_I &= \arcsin\left(n_1/n_2\right) \\ &= \arcsin\left(1/n_2\right) && \text{(for air)} \\ &= \arctan\left[\frac{1}{\left(n_2^2-1\right)^{1/2}}\right] && \text{(for air)}\end{aligned}$$

Equating either $\sin\left(\theta_B\right)=\sin\left(\theta_I\right)$ or $\tan\left(\theta_B\right)=\tan\left(\theta_I\right)$ for air yields a quadratic equation in n_2^2:

$$n_2^4-n_2^2-1=0,$$

from which the necessarily real and positive refractive index is $n_2=\left[\frac{1+5^{1/2}}{2}\right]^{1/2}\approx 1.27$.